Statistics for K_H-tables

For a number field K let K_1 be the Hilbert class field of $K_0 := K$, let K_2 be the Hilbert class field of K_1, ... and let K_H be the maximal soluble unramified extension of K. The length of the class field tower is the minimal integer i such that $K_H = K_i$ or ∞.

There are 3043 real quadratic number fields K with discriminant $d_K < 10000$. The following gives the Galois group $\mathrm{Gal}(K_H/\mathbb{Q})$ for the 225 K with length of class field tower greater then 1. The results depend on GRH.

G_n denotes the group $\langle x, y, z \,|\, x^{2\cdot n} = y^2 = z^2 = 1, x^y = x, x^z = x^{-1}, y^z = x^n y\rangle$ and IdSG the Identification of the group in the Small Group Library of GAP (`http://www.gap-system.org`).

d_K	$(K_H : \mathbb{Q})$	H	Gal	IdSG
520	16	2	G_2	[16,13]
1145	24	2	G_3	[24,14]
1160	16	2	$C_2 \times \mathbb{D}_{2\cdot 4}$	[16,11]
1285	12	2	$\mathbb{D}_{2\cdot 6}$	[12,4]
1580	12	2	$\mathbb{D}_{2\cdot 6}$	[12,4]
1605	12	2	$\mathbb{D}_{2\cdot 6}$	[12,4]
1640	16	2	$C_2 \times \mathbb{D}_{2\cdot 4}$	[16,11]
1740	32	2	G_4	[32,42]
1785	32	2	$(C_2 \times \mathbb{D}_{2\cdot 4}) \rtimes C_2$	[32,43]
1820	16	2	G_2	[16,13]
1832	12	2	$\mathbb{D}_{2\cdot 6}$	[12,4]
1885	16	2	$C_2 \times \mathbb{D}_{2\cdot 4}$	[16,11]
1957	24	2	$\mathbb{S}_4$	[24,12]
2005	40	2	G_5	[40,8]
2056	24	2	G_3	[24,14]
2120	16	2	G_2	[16,13]
2345	12	2	$\mathbb{D}_{2\cdot 6}$	[12,4]
2365	12	2	$\mathbb{D}_{2\cdot 6}$	[12,4]
2405	16	2	G_2	[16,13]
2460	16	2	$C_2 \times \mathbb{D}_{2\cdot 4}$	[16,11]
2465	32	2	G_4	[32,42]
2568	12	2	$\mathbb{D}_{2\cdot 6}$	[12,4]
2652	16	2	$C_2 \times \mathbb{D}_{2\cdot 4}$	[16,11]
2748	12	2	$\mathbb{D}_{2\cdot 6}$	[12,4]
2777	48	3	$GL(2,3)$	[48,29]
2840	12	2	$\mathbb{D}_{2\cdot 6}$	[12,4]
2856	16	2	$C_2 \times \mathbb{D}_{2\cdot 4}$	[16,11]
2885	28	2	$\mathbb{D}_{2\cdot 14}$	[28,3]
2977	12	2	$\mathbb{D}_{2\cdot 6}$	[12,4]
3045	16	2	$C_2 \times \mathbb{D}_{2\cdot 4}$	[16,11]
3084	12	2	$\mathbb{D}_{2\cdot 6}$	[12,4]
3145	16	2	G_2	[16,13]
3180	16	2	G_2	[16,13]
3208	20	2	$\mathbb{D}_{2\cdot 10}$	[20,4]

d_K	$(K_H : \mathbb{Q})$	H	Gal	IdSG
3341	24	2	G_3	[24,14]
3445	16	2	$C_2 \times \mathbb{D}_{2\cdot 4}$	[16,11]
3480	32	2	$(C_4 \times V_4) \rtimes C_2$	[32,28]
3485	24	2	$V_4 \times \mathbb{S}_3$	[24,14]
3560	16	2	$C_2 \times \mathbb{D}_{2\cdot 4}$	[16,11]
3660	16	2	$C_2 \times \mathbb{D}_{2\cdot 4}$	[16,11]
3665	12	2	$\mathbb{D}_{2\cdot 6}$	[12,4]
3720	16	2	G_2	[16,13]
3752	12	2	$\mathbb{D}_{2\cdot 6}$	[12,4]
3784	12	2	$\mathbb{D}_{2\cdot 6}$	[12,4]
3805	24	2	G_3	[24,14]
3885	16	2	$C_2 \times \mathbb{D}_{2\cdot 4}$	[16,11]
3893	12	2	$\mathbb{D}_{2\cdot 6}$	[12,4]
3965	32	2	$(C_2 \times \mathbb{D}_{2\cdot 4}) \rtimes C_2$	[32,43]
3981	24	2	$\mathbb{S}_4$	[24,12]
4040	16	2	$C_2 \times \mathbb{D}_{2\cdot 4}$	[16,11]
4060	16	2	$C_2 \times \mathbb{D}_{2\cdot 4}$	[16,11]
4085	20	2	$\mathbb{D}_{2\cdot 10}$	[20,4]
4108	12	2	$\mathbb{D}_{2\cdot 6}$	[12,4]
4173	24	2	G_3	[24,14]
4264	16	2	$C_2 \times \mathbb{D}_{2\cdot 4}$	[16,11]
4305	32	2	$(C_4 \times V_4) \rtimes C_2$	[32,28]
4369	12	2	$\mathbb{D}_{2\cdot 6}$	[12,4]
4380	32	2	G_4	[32,42]
4460	12	2	$\mathbb{D}_{2\cdot 6}$	[12,4]
4485	16	2	$C_2 \times \mathbb{D}_{2\cdot 4}$	[16,11]
4488	32	2	$(C_4 \times V_4) \rtimes C_2$	[32,28]
4505	32	2	G_4	[32,42]
4520	64	2	G_8	[64,189]
4524	16	2	$C_2 \times \mathbb{D}_{2\cdot 4}$	[16,11]
4616	56	2	G_7	[56,7]
4620	32	2	$C_2 \times G_2$	[32,48]
4641	48	2	$\mathbb{S}_3 \times \mathbb{D}_{2\cdot 4}$	[48,38]
4745	16	2	G_2	[16,13]
4785	16	2	$C_2 \times \mathbb{D}_{2\cdot 4}$	[16,11]
4809	12	2	$\mathbb{D}_{2\cdot 6}$	[12,4]
4812	20	2	$\mathbb{D}_{2\cdot 10}$	[20,4]
4845	16	2	G_2	[16,13]
4872	16	2	G_2	[16,13]
4920	16	2	$C_2 \times \mathbb{D}_{2\cdot 4}$	[16,11]
4965	12	2	$\mathbb{D}_{2\cdot 6}$	[12,4]
5045	56	2	G_7	[56,7]
5080	12	2	$\mathbb{D}_{2\cdot 6}$	[12,4]
5213	20	2	$\mathbb{D}_{2\cdot 10}$	[20,4]
5304	16	2	$C_2 \times \mathbb{D}_{2\cdot 4}$	[16,11]
5340	16	2	$C_2 \times \mathbb{D}_{2\cdot 4}$	[16,11]

d_K	$(K_H : \mathbb{Q})$	H	Gal	IdSG
5365	32	2	G_4	[32,42]
5372	12	2	$\mathbb{D}_{2\cdot 6}$	[12,4]
5397	48	3	$C_2 \times \mathbb{S}_4$	[48,48]
5457	12	2	$\mathbb{D}_{2\cdot 6}$	[12,4]
5465	20	2	$\mathbb{D}_{2\cdot 10}$	[20,4]
5480	16	2	$C_2 \times \mathbb{D}_{2\cdot 4}$	[16,11]
5496	12	2	$\mathbb{D}_{2\cdot 6}$	[12,4]
5505	12	2	$\mathbb{D}_{2\cdot 6}$	[12,4]
5512	16	2	$C_2 \times \mathbb{D}_{2\cdot 4}$	[16,11]
5576	48	2	$\mathbb{S}_3 \times \mathbb{D}_{2\cdot 4}$	[48,38]
5628	24	2	$V_4 \times \mathbb{S}_3$	[24,14]
5645	36	2	$\mathbb{D}_{2\cdot 18}$	[36,4]
5676	24	2	$V_4 \times \mathbb{S}_3$	[24,14]
5740	16	2	$C_2 \times \mathbb{D}_{2\cdot 4}$	[16,11]
5785	16	2	$C_2 \times \mathbb{D}_{2\cdot 4}$	[16,11]
5820	16	2	$C_2 \times \mathbb{D}_{2\cdot 4}$	[16,11]
5864	12	2	$\mathbb{D}_{2\cdot 6}$	[12,4]
5945	16	2	$C_2 \times \mathbb{D}_{2\cdot 4}$	[16,11]
6060	32	2	$C_2 \times \mathbb{D}_{2\cdot 8}$	[32,39]
6088	72	2	$(C_6 \times \mathbb{S}_3) \rtimes C_2$	[72,23]
6097	84	2	$\mathbb{S}_3 \times \mathbb{D}_{2\cdot 7}$	[84,8]
6105	16	2	$C_2 \times \mathbb{D}_{2\cdot 4}$	[16,11]
6145	24	2	G_3	[24,14]
6149	48	3	$C_2 \times \mathbb{S}_4$	[48,48]
6168	12	2	$\mathbb{D}_{2\cdot 6}$	[12,4]
6188	16	2	$C_2 \times \mathbb{D}_{2\cdot 4}$	[16,11]
6216	32	2	G_4	[32,42]
6280	24	2	$V_4 \times \mathbb{S}_3$	[24,14]
6285	12	2	$\mathbb{D}_{2\cdot 6}$	[12,4]
6344	16	2	$C_2 \times \mathbb{D}_{2\cdot 4}$	[16,11]
6380	16	2	$C_2 \times \mathbb{D}_{2\cdot 4}$	[16,11]
6405	16	2	$C_2 \times \mathbb{D}_{2\cdot 4}$	[16,11]
6412	12	2	$\mathbb{D}_{2\cdot 6}$	[12,4]
6440	32	2	$(C_2 \times \mathbb{D}_{2\cdot 4}) \rtimes C_2$	[32,43]
6460	16	2	$C_2 \times \mathbb{D}_{2\cdot 4}$	[16,11]
6485	44	2	$\mathbb{D}_{2\cdot 22}$	[44,3]
6520	12	2	$\mathbb{D}_{2\cdot 6}$	[12,4]
6536	20	2	$\mathbb{D}_{2\cdot 10}$	[20,4]
6540	16	2	$C_2 \times \mathbb{D}_{2\cdot 4}$	[16,11]
6565	64	2	$((C_4 \times C_4) \rtimes C_2) \rtimes C_2$	[64,134]
6636	24	2	$V_4 \times \mathbb{S}_3$	[24,14]
6641	12	2	$\mathbb{D}_{2\cdot 6}$	[12,4]
6765	16	2	$C_2 \times \mathbb{D}_{2\cdot 4}$	[16,11]
6809	72	2	$(V_4 \rtimes C_9) \rtimes C_2$	[72,15]
6817	20	2	$\mathbb{D}_{2\cdot 10}$	[20,4]
6865	12	2	$\mathbb{D}_{2\cdot 6}$	[12,4]

d_K	$(K_H:\mathbb{Q})$	H	Gal	IdSG
6888	16	2	$C_2 \times \mathbb{D}_{2\cdot 4}$	[16,11]
6920	24	2	$V_4 \times \mathbb{S}_3$	[24,14]
6924	56	2	G_7	[56,7]
6965	20	2	$\mathbb{D}_{2\cdot 10}$	[20,4]
7053	24	2	$\mathbb{S}_4$	[24,12]
7145	20	2	$\mathbb{D}_{2\cdot 10}$	[20,4]
7176	16	2	$C_2 \times \mathbb{D}_{2\cdot 4}$	[16,11]
7180	12	2	$\mathbb{D}_{2\cdot 6}$	[12,4]
7196	12	2	$\mathbb{D}_{2\cdot 6}$	[12,4]
7208	16	2	$C_2 \times \mathbb{D}_{2\cdot 4}$	[16,11]
7240	16	2	$C_2 \times \mathbb{D}_{2\cdot 4}$	[16,11]
7293	16	2	$C_2 \times \mathbb{D}_{2\cdot 4}$	[16,11]
7320	16	2	$C_2 \times \mathbb{D}_{2\cdot 4}$	[16,11]
7384	12	2	$\mathbb{D}_{2\cdot 6}$	[12,4]
7445	12	2	$\mathbb{D}_{2\cdot 6}$	[12,4]
7453	108	2	$(((C_3 \times C_3) \rtimes C_3) \rtimes C_2) \rtimes C_2$	[108,17]
7501	28	2	$\mathbb{D}_{2\cdot 14}$	[28,3]
7537	24	2	$\mathbb{S}_4$	[24,12]
7545	12	2	$\mathbb{D}_{2\cdot 6}$	[12,4]
7557	24	2	G_3	[24,14]
7565	16	2	$C_2 \times \mathbb{D}_{2\cdot 4}$	[16,11]
7685	16	2	$C_2 \times \mathbb{D}_{2\cdot 4}$	[16,11]
7752	16	2	$C_2 \times \mathbb{D}_{2\cdot 4}$	[16,11]
7880	48	2	G_6	[48,37]
7917	16	2	$C_2 \times \mathbb{D}_{2\cdot 4}$	[16,11]
7944	12	2	$\mathbb{D}_{2\cdot 6}$	[12,4]
7973	12	2	$\mathbb{D}_{2\cdot 6}$	[12,4]
7980	64	2	$(C_2 \times \mathbb{D}_{2\cdot 8}) \rtimes C_2$	[64,257]
8005	112	2	$C_7 \rtimes \mathbb{D}_{2\cdot 8}$	[112,14]
8041	12	2	$\mathbb{D}_{2\cdot 6}$	[12,4]
8069	192	3	$(((C_4 \times C_2) \rtimes C_4) \rtimes C_3) \rtimes C_2$	[192,181]
8072	28	2	$\mathbb{D}_{2\cdot 14}$	[28,3]
8120	16	2	$C_2 \times \mathbb{D}_{2\cdot 4}$	[16,11]
8205	20	2	$\mathbb{D}_{2\cdot 10}$	[20,4]
8245	16	2	G_2	[16,13]
8265	16	2	$C_2 \times \mathbb{D}_{2\cdot 4}$	[16,11]
8268	32	2	$C_2 \times \mathbb{D}_{2\cdot 8}$	[32,39]
8360	16	2	$C_2 \times \mathbb{D}_{2\cdot 4}$	[16,11]
8364	48	2	G_6	[48,37]
8385	16	2	$C_2 \times \mathbb{D}_{2\cdot 4}$	[16,11]
8421	20	2	$\mathbb{D}_{2\cdot 10}$	[20,4]
8473	48	2	$C_3 \rtimes \mathbb{D}_{2\cdot 8}$	[48,15]
8481	12	2	$\mathbb{D}_{2\cdot 6}$	[12,4]
8520	16	2	G_2	[16,13]
8540	48	2	$\mathbb{S}_3 \times \mathbb{D}_{2\cdot 4}$	[48,38]
8584	16	2	G_2	[16,13]

d_K	$(K_H : \mathbb{Q})$	H	Gal	IdSG
8585	32	2	$(C_2 \times \mathbb{D}_{2\cdot 4}) \rtimes C_2$	[32,43]
8680	48	2	$\mathbb{S}_3 \times \mathbb{D}_{2\cdot 4}$	[48,38]
8744	20	2	$\mathbb{D}_{2\cdot 10}$	[20,4]
8745	48	2	$(C_4 \times \mathbb{S}_3) \rtimes C_2$	[48,41]
8760	16	2	$C_2 \times \mathbb{D}_{2\cdot 4}$	[16,11]
8780	20	2	$\mathbb{D}_{2\cdot 10}$	[20,4]
8789	48	2	$C_2 \times \mathbb{S}_4$	[48,48]
8796	12	2	$\mathbb{D}_{2\cdot 6}$	[12,4]
8805	28	2	$\mathbb{D}_{2\cdot 14}$	[28,3]
8808	12	2	$\mathbb{D}_{2\cdot 6}$	[12,4]
8840	128	2	$(C_4 \times S\mathbb{D}_{8\cdot 2}) \rtimes C_2$	[128,1851]
8845	16	2	$C_2 \times \mathbb{D}_{2\cdot 4}$	[16,11]
8860	12	2	$\mathbb{D}_{2\cdot 6}$	[12,4]
8904	16	2	G_2	[16,13]
8940	16	2	$C_2 \times \mathbb{D}_{2\cdot 4}$	[16,11]
8988	24	2	$V_4 \times \mathbb{S}_3$	[24,14]
9020	16	2	$C_2 \times \mathbb{D}_{2\cdot 4}$	[16,11]
9032	72	2	G_9	[72,8]
9048	16	2	$C_2 \times \mathbb{D}_{2\cdot 4}$	[16,11]
9061	24	2	$V_4 \times \mathbb{S}_3$	[24,14]
9132	12	2	$\mathbb{D}_{2\cdot 6}$	[12,4]
9160	48	2	$C_2 \times G_3$	[48,43]
9164	12	2	$\mathbb{D}_{2\cdot 6}$	[12,4]
9240	32	2	$(C_2 \times \mathbb{D}_{2\cdot 4}) \rtimes C_2$	[32,49]
9301	48	3	$GL(2,3)$	[48,29]
9309	12	2	$\mathbb{D}_{2\cdot 6}$	[12,4]
9345	32	2	$(C_4 \times V_4) \rtimes C_2$	[32,28]
9384	16	2	$C_2 \times \mathbb{D}_{2\cdot 4}$	[16,11]
9389	12	2	$\mathbb{D}_{2\cdot 6}$	[12,4]
9420	24	2	$V_4 \times \mathbb{S}_3$	[24,14]
9480	16	2	G_2	[16,13]
9485	20	2	$\mathbb{D}_{2\cdot 10}$	[20,4]
9505	84	2	$\mathbb{S}_3 \times \mathbb{D}_{2\cdot 7}$	[84,8]
9509	12	2	$\mathbb{D}_{2\cdot 6}$	[12,4]
9512	32	2	G_4	[32,42]
9516	64	2	$(C_8 \times V_4) \rtimes C_2$	[64,147]
9529	12	2	$\mathbb{D}_{2\cdot 6}$	[12,4]
9624	20	2	$\mathbb{D}_{2\cdot 10}$	[20,4]
9640	32	2	$(C_2 \times \mathbb{D}_{2\cdot 4}) \rtimes C_2$	[32,43]
9645	12	2	$\mathbb{D}_{2\cdot 6}$	[12,4]
9656	12	2	$\mathbb{D}_{2\cdot 6}$	[12,4]
9660	32	2	$(C_2 \times \mathbb{D}_{2\cdot 4}) \rtimes C_2$	[32,49]
9685	24	2	$V_4 \times \mathbb{S}_3$	[24,14]
9724	16	2	$C_2 \times \mathbb{D}_{2\cdot 4}$	[16,11]
9768	32	2	$(C_2 \times \mathbb{D}_{2\cdot 4}) \rtimes C_2$	[32,43]
9785	48	3	$C_2 \times \mathbb{S}_4$	[48,48]

d_K	$(K_H : \mathbb{Q})$	H	Gal	IdSG
9804	80	2	$\mathbb{D}_{2\cdot 5} \times \mathbb{D}_{2\cdot 4}$	[80,39]
9809	28	2	$\mathbb{D}_{2\cdot 14}$	[28,3]
9832	12	2	$\mathbb{D}_{2\cdot 6}$	[12,4]
9893	12	2	$\mathbb{D}_{2\cdot 6}$	[12,4]
9933	24	2	$V_4 \times \mathbb{S}_3$	[24,14]
9960	32	2	$(C_2 \times \mathbb{D}_{2\cdot 4}) \rtimes C_2$	[32,43]
9980	60	2	$\mathbb{S}_3 \times \mathbb{D}_{2\cdot 5}$	[60,8]